

PHASE OSCILLATOR NETWORK MODELS OF BRAIN DYNAMICS

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Abstract. Networks of periodically firing neurons can be modelled as networks of coupled phase oscillators, each oscillator being described by a single angular variable. Networks of two types of neural phase oscillators are analysed here: the theta neuron and the Winfree oscillator. By taking the limit of an infinite number of neurons and using the Ott/Antonsen ansatz, we derive and then numerically analyse “neural field” type differential equations which govern the evolution of macroscopic order parameter-like quantities. The mathematical framework presented here allows one efficiently simulate such networks, and to investigate the effects of changing the structure of a network of neurons, or the parameters of such networks.

1. Introduction

It is well established that a single neuron can fire a periodic train of action potent o

ones. The study of oscillations in neuroscience is a large topic [55, 56, 20, 10, 54] and

identical. Thus we can determine the asymptotic dynamics of (6) by assuming that F is given by (9). It is helpful to introduce the complex order parameter, as considered by Kuramoto in the context of coupled phase oscillators [30, 53]

$$(10) \quad z(t) = \int_{-\infty}^{\infty} \int_0^{2\pi} F(I, \theta, t) e^{i\theta} d\theta dI.$$

The quantity z can be thought of as the expected value of $e^{i\theta}$. Substituting the ansatz (9)

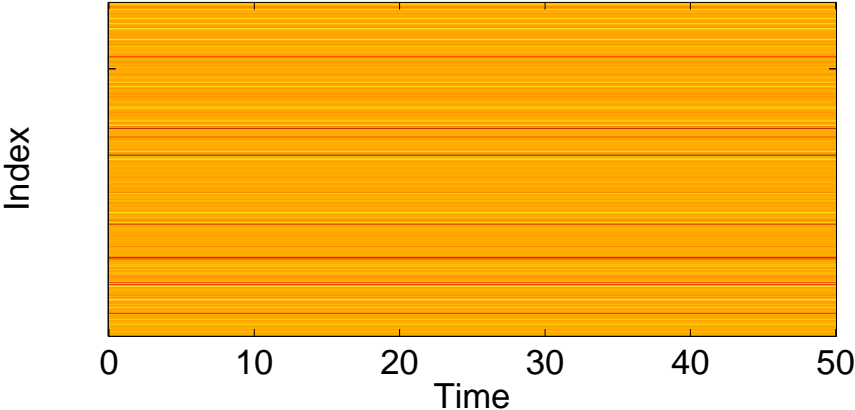
of the infinite network. This pair of equations was studied with $\tau = 0$, i.e. instantaneous synapses, by [39]. For a physical interpretation of $z \in \mathbb{C}$, write $z(t) = r(t)e^{i\psi(t)}$. Integrating (9) over θ we obtain the probability density function

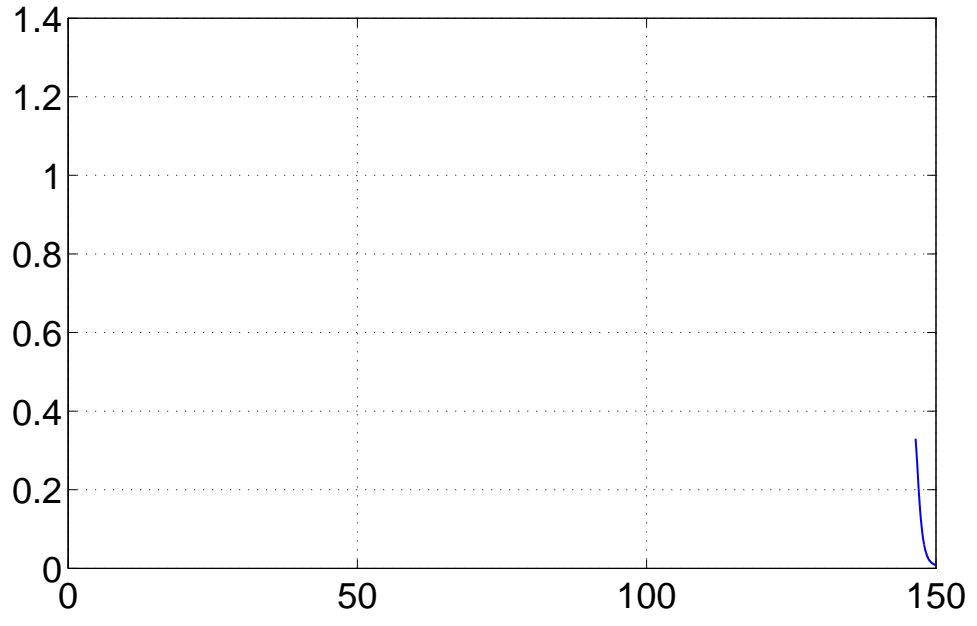
$$(18) \quad p(\theta, t) = \frac{1 - r^2(t)}{2 \{1 - 2r(t) \cos[\theta - \psi(t)] + r^2(t)\}}$$

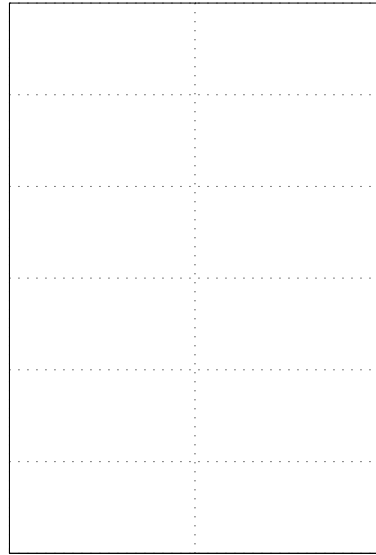
which is a unimodal function of θ with maximum at $\theta = \psi(t)$, and whose sharpness is governed by the value of r [35, 34]. Alternatively, we follow [42] and define

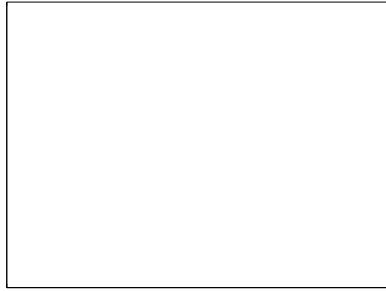
$$(19) \quad w = \frac{1 - \bar{z}}{1 + \bar{z}} = \frac{1 + 2ir \sin \theta - r^2}{1 + 2r \cos \theta + r^2}.$$

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and have

$$(42) \quad z(t) = \int_{-\infty}^{\infty} \int_0^{2\pi} F(\theta, t) e^{i\theta} d\theta d\theta .$$

equations are then

$$\frac{z(x, t)}{t} = \frac{R(x, t)e^{-i\beta}}{2} + (i$$

References

[23] Boris Gutkin. Theta-neuron model. In Dieter Jaeger and Ranu Jung, editors,

